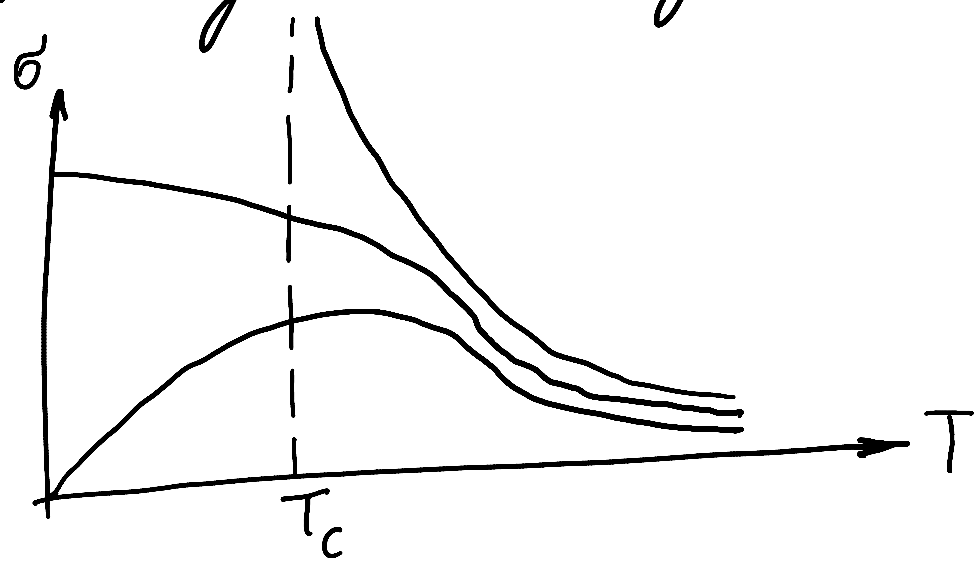
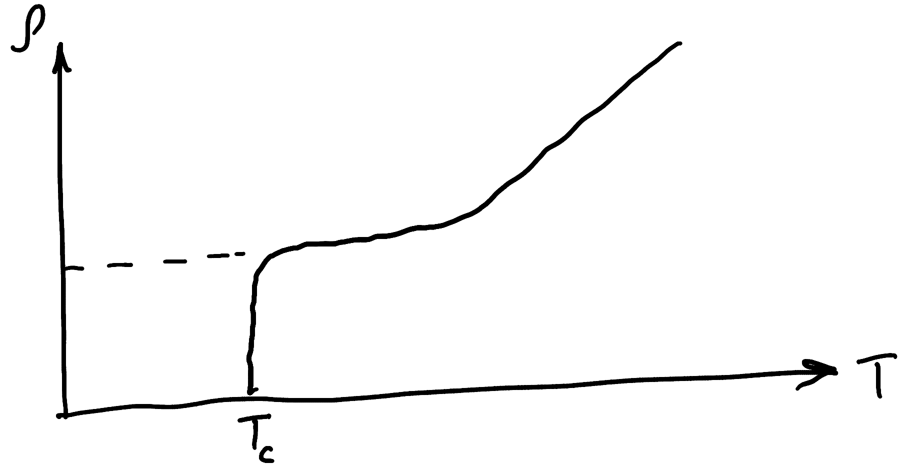
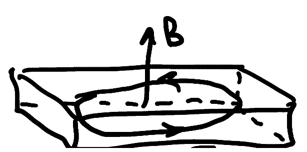
Superconductivity

Vanishing of resistivity





*) Meissner effect
(= Meissner-Ochsenfeld effect)
– the vanishing of magnetic field inside of a superconductor
(in fact, applies to <u>type I</u> superconductors)



Note: this is not a consequence of vanishing resistivity

Note: this is not a theory of vanishing resistivity

$\mathcal{E} = \frac{1}{c}\frac{\partial \varphi}{\partial t}$

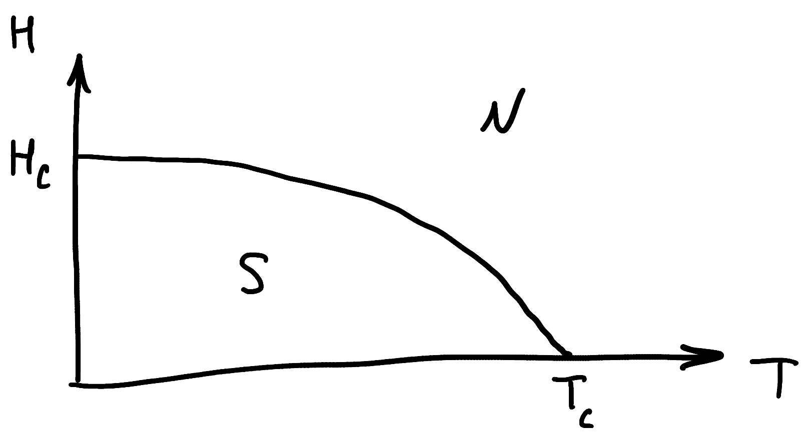
voltage induced in a closed loop = electromotive force

One needs $\mathcal{E} = 0$.
Otherwise, $I = \frac{\mathcal{E}}{R} \to \infty$
Thus, the system must induce current to screen the change of the external magnetic field

Thus $\varphi = \text{const}$, so long as $\sigma = \infty$

However, in a <u>superconductor</u>, unlike an ideal metal, $\varphi = 0$ – a thermodynamic state

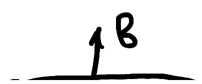
*) Superconductivity is destroyed by a sufficiently strong magnetic field



$$H_c(T) = H_c\left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

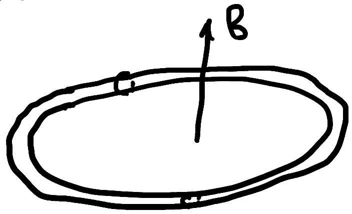
*) <u>Quantisation of magnetic flux</u>

If we make a (sufficiently thick) superconductive ring



$n \quad \pi\hbar c$

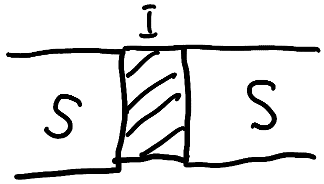
superconducting



$$\varphi_0 = \frac{\pi \hbar c}{e}$$

(Twice the „electronic" flux $\varphi_0^e = \frac{2\pi \hbar c}{e}$)

*) Josephson effect



I - current flowing without resistance if $I < I_c$

If $I > I_c$; there will be some voltage V with an oscillating component

$$\hbar \omega = 2eV$$

Superconductors

Type I

(All element SC (Al, Cd, Ga, ...) except Nb)

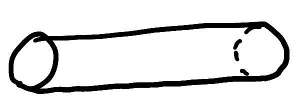
Meissner effect

Type II

Alloys

Magnetic field partially penetrates for $H_{c1} < H < H_{c2}$

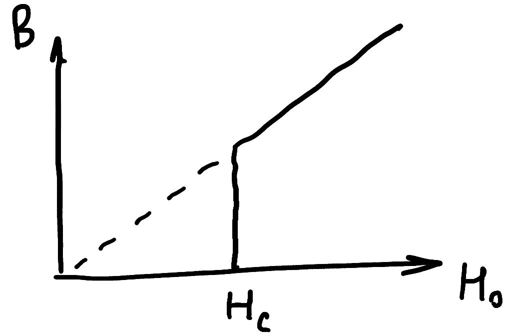
Consider Type I superconductors



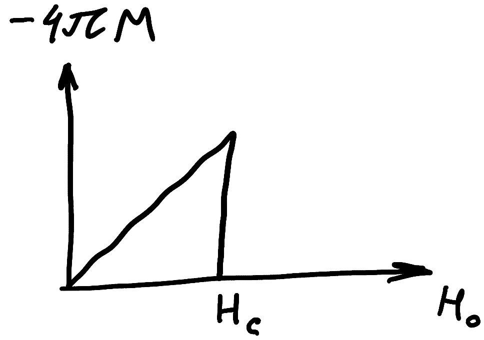
Magnetisation

B

$-4\pi M$



Magnetisation

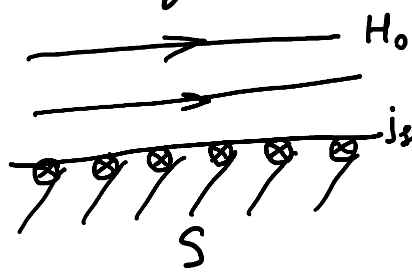


$$\vec{B} = \vec{H}_0 + 4\pi \vec{M}$$

Inside a superconductor there is no magnetic field, and on it's surface there are no magnetic charges ($\operatorname{div} \vec{B} = 0$)

→ The normal magnetic field vanishes

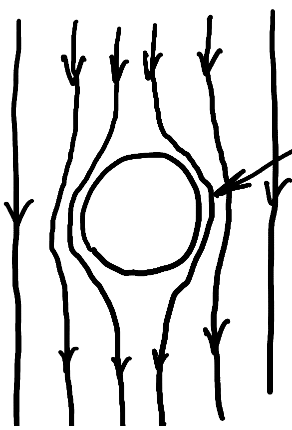
→ The magnetic field is always tangential



$$j_{surf} = \frac{c}{4\pi} H_0$$

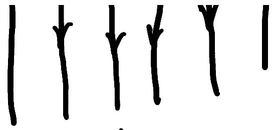
$$\vec{j}_{surf} = \frac{c}{4\pi} \vec{n} \times \vec{H}_0$$

Penetration of magnetic field into a superconductor



Magnetic field here is stronger and will reach H_c before H_0 reaches H_c

The ball may be in a mixed



H_0

The ball may be in a mixed state

$$H_c \to H_m = \frac{H_0}{1-n} \qquad (n<1)$$

n - demagnetising factor

Thermodynamics

In type I superconductors $\vec{M} = -\frac{\vec{H}}{4\pi}$

when $\vec{H}$ changes, $\vec{H} \to \vec{H} + d\vec{H}$, it does the amount of work

$$-\vec{M}\, d\vec{H} = \frac{1}{4\pi} \vec{H}\, d\vec{H}$$

If $F(H)$ is the free energy of the superconductor,

$$F(H) - F(0) = \frac{H^2}{8\pi}$$

If it is favourable to switch to the normal state, the superconductor will

Thus, $$\boxed{F_n = F_s + \frac{H_c^2}{8\pi}} \qquad (*)$$

Find heat capacitance from the free energy

$$F = U - TS$$

$$dF = dU - TdS - SdT = -dW_b - SdT$$

$$dF = dU - TdS - SdT = -\underset{\text{work}}{dW} - SdT$$

$$\rightarrow S = -\left(\frac{\partial F}{\partial T}\right)_W$$

Use formula (*) above

$$S_s - S_n = \frac{H_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)_W$$

Several consequences:

o) Because $S=0$ for $T=0$ (Nernst theorem)

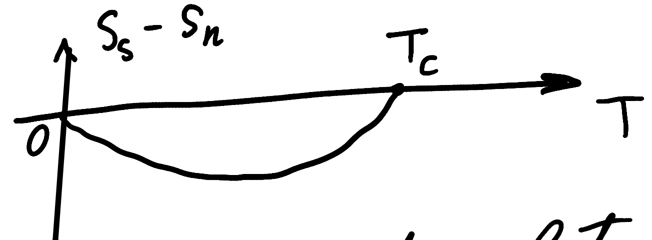
$$\left(\frac{\partial H_c}{\partial T}\right)_{T=0} = 0$$

o) $\frac{\partial H_c}{\partial T} < 0$ – experimental fact

$$\rightarrow S_s < S_n$$

→ Superconductivity is a more ordered state than normal metal

o) For $T = T_c$ $H_c = 0$ and $S_s = S_n$



→ The transition between S and N occurs without absorbing or emitting heat (at T_c)

On the other hang, if $T < T_c$ from N to S

On the other hand, if [the field increases] and the system goes from N to S due to changing the magnetic field, then it releases heat.

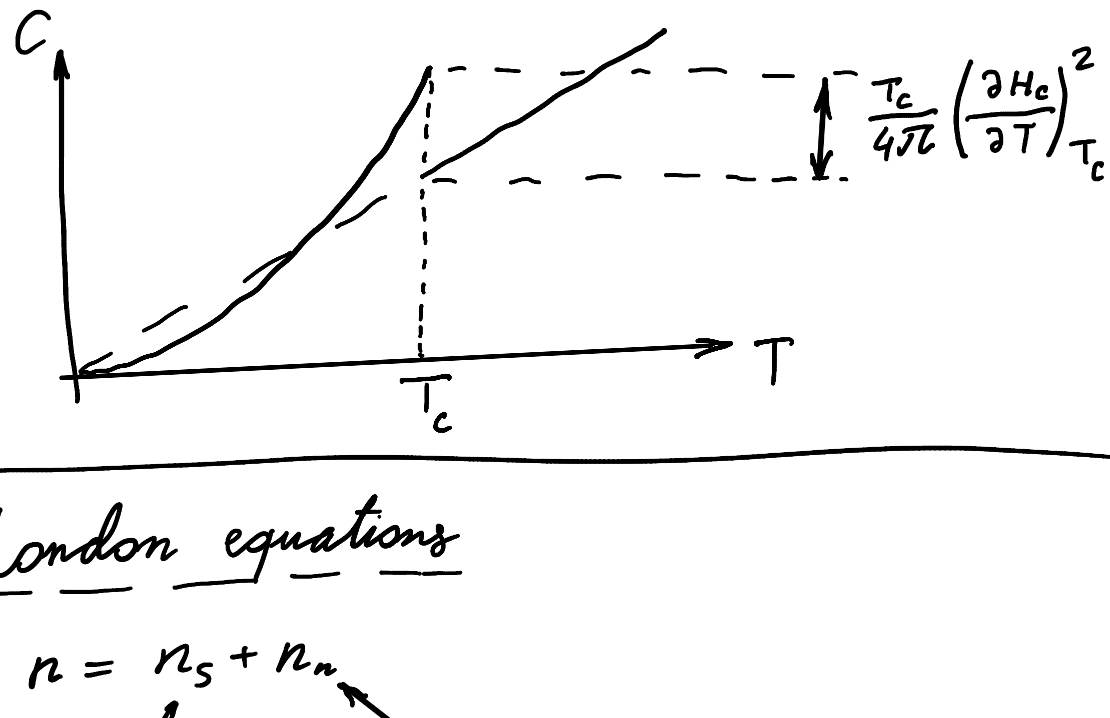
Heat capacitance

$$C = T\frac{\partial S}{\partial T}$$

$$C_s - C_n = \frac{T}{4\pi}\left[\left(\frac{\partial H_c}{\partial T}\right)^2 + H_c\frac{\partial^2 H_c}{\partial T^2}\right]$$

$$\boxed{C_s - C_n = \frac{T_c}{4\pi}\left(\frac{\partial H_c}{\partial T}\right)^2_{T_c}}$$ – Rutgers formula

– jump of the heat capacitance at the S-N transition



London equations

$$n = n_s + n_n$$

„Superconductive" electrons   „Normal" electrons

$$n_s m \frac{d\vec{v}_s}{dt} = n_s e \vec{E}$$

1-st London equation

$$\boxed{\vec{E} = \frac{d}{dt}(\Lambda \vec{j}_s)}, \text{ where } \Lambda = \frac{m}{n_s e^2}$$

and $\vec{j}_s = n_s e \vec{v}_s$

The kinetic energy of the „super" electrons:

$$W_{kin} = \frac{n_s m v_s^2}{2} = \frac{m j_s^2}{2 n_s e^2}$$

Maxwell equation $\text{rot } \vec{H} = \frac{4\pi}{c} \vec{j}_s$

$$\rightarrow \quad W_{kin} = \frac{\lambda^2}{8\pi} (\text{rot } \vec{H})^2$$

where $\lambda^2 = \frac{m c^2}{4\pi n_s e^2}$

The density of the magnetic and kinetic energies:

$$F_{sH} = F_{so} + \frac{1}{8\pi} \int \left[H^2 + \lambda^2 (\text{rot } \vec{H})^2 \right] dV$$

Consider a variation of this free energy

$$\delta F_s = \frac{1}{8\pi} \int (2\vec{H}\,\delta\vec{H} + 2\lambda^2 \text{rot } \vec{H} \; \text{rot } \delta\vec{H})\, dV$$

Use that $\underbrace{\vec{a}}_{=\text{rot } \hat{H}} \text{rot } \underbrace{\vec{b}}_{=\delta\vec{H}} = \vec{b} \text{ rot } \vec{a} - \text{div } \vec{a} \times \vec{b}$

$$\delta F_s = \frac{1}{4\pi} \int_V \left[\vec{H} + \lambda^2 \text{rot rot } \vec{H} \right] \delta\vec{H} - \frac{1}{4\pi} \underbrace{\int_V \text{div}(\text{rot } \vec{H} \times \delta\vec{H})}_{"} = 0$$

$$-\frac{1}{4\pi}\int_V dV\,(\ldots)$$

$$\| \quad \int_{\text{Surface}} \text{rot}\,\vec H\times\delta\vec H\; d\vec S = 0$$

Because the field on the surface is fixed when finding the variational derivative

Then $\boxed{\vec H + \lambda^2\,\text{rot}\,\text{rot}\,\vec H = 0}$ – 2nd London equation

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If we use $\text{rot}\,\vec H = \frac{4\pi}{c}\vec j$

$$\text{rot}\,\vec A + \frac{4\pi\lambda^2}{c}\,\text{rot}\,\vec j = 0$$

(*)

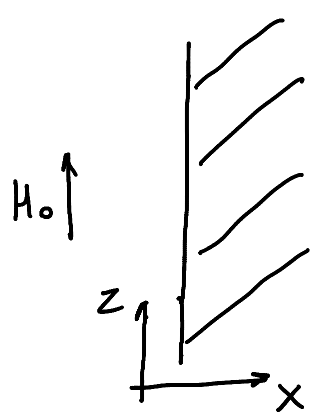
$$\vec j = -\frac{c}{4\pi\lambda^2}\vec A$$

if we choose gauge $\text{div}\,\vec A = 0$, $\vec A\vec n = 0$

$$\Lambda = \frac{4\pi\lambda^2}{c^2}$$

(*) follows also from the first London equation

Penetration depth



$$\vec H + \lambda^2\,\text{rot}\,\text{rot}\,\vec H = 0$$

$$H + \lambda^2\frac{d^2H}{dx^2} = 0$$

$$H = H_0\, e^{-\frac{x}{\lambda}}$$

$$\lambda = \left(\frac{mc^2}{4\pi n_s e^2}\right)^{\frac{1}{2}}$$ – penetration depth

$$j_s = \frac{cH_0}{4\pi\lambda}\, e^{-\frac{x}{\lambda}}$$

$$j_s = \frac{c H_0}{4\pi\lambda} e^{-\lambda}$$

Empiric formula:

$$\lambda(T) = \frac{\lambda(0)}{\left(1-\left(\frac{T}{T_c}\right)^4\right)^{\frac{1}{2}}}$$

Flux quantisation

Charge carriers in a superconductor are Cooper pairs. They all are on the same level = form a condensate

Their wavefunction is $\psi(r) = \left(\frac{n_s}{2}\right)^{\frac{1}{2}} e^{i\theta(r)}$

The momentum of a particle with the mass $2m$ and charge $2e$

$$\hbar \vec{\nabla}\theta = 2m\vec{v}_s + \frac{2e}{c}\vec{A}$$

Taking into account that $\vec{j}_s = n_s e \vec{v}_s$,

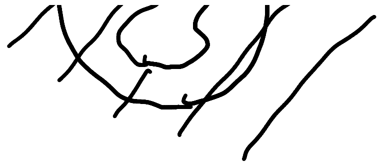
$$\vec{j}_s = \frac{1}{c\Lambda}\left(\frac{\varphi_0}{2\pi}\vec{\nabla}\theta - \vec{A}\right)$$

where $\varphi_0 = \frac{\pi\hbar c}{|e|}$

Consider a contor encompassing a cavity in an SC

Along the contour $j_c = 0$

$$\rightarrow \varphi = \varphi_0 \cdot n, \quad n \in \mathbb{Z}$$



$\rightarrow \varphi = \varphi_0 \cdot n \ , \ n \in \mathbb{Z}$